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# Investigations of resonant systems in celestial mechanics

**Theses of Dissertation**

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# 1 Introduction

Resonances play an important role in the dynamics of our solar system. The long term orbital evolution of the minor and major bodies depend on mean motion and secular resonances. Mean motion resonances occur between celestial bodies when their orbital periods are commensurate, their relation can be given as the quotient of small whole numbers.

The number of celestial bodies moving on resonant orbits is larger than it would be justified by a random distribution, their existence is the result of long term dynamical evolution. Attempts have been made to identify a precise resonant structure for the solar system [1], but the deviations from the precise commensurabilities are too large to have influence [2].

Only a few exact mean motion resonances are known between planets, dwarf planets and moons, in much greater number are affected asteroids, planetary rings and Kuiper belt objects. Small bodies are important components of the solar system. individually they do not influence the planets orbits in general, but in the early stage of planetary system formation they played an important role in the orbit's evolution. Two areas of the solar system, that are rich in asteroids are the main asteroid belt, and the Kuiper belt. The effect of resonant phenomena can be followed up on the distribution of asteroids located there. In this dissertation I study mean motion resonances in these two regions.

The Trojan asteroids make up a large group of the main asteroid belt. Coorbital with Jupiter, they move in 1:1 mean motion resonance with it, near the $L_4$ and $L_5$ Lagrange points of the system. Besides the fact, that their configuration is dynamically very interesting, stability investigations and the analysis of their motion is an important task, due to their large number and proximity. Their stability has been a concern to researcher for a long time [3, 4, 7]. It is shown that systems with particular mass parameter ($\mu$) and eccentricity ($e$) values are stable, and others are not. Stable and unstable $\mu, e$ regions have been determined, their structure is investigated [4, 8, 9], frequencies and resonances are studied [5, 6].

The resonant structure of the asteroid-rich regions play an important role in the orbital evolution of asteroids. Primarily low-order resonances are of great importance (consider the case of Kirkwood gaps), although the effect of higher-order resonances is also significant, particularly on high eccentricities. Researches have shown relationship between the locations of high order resonances and distribution and number of members in asteroid families [10].

Trans-Neptunian objects are also related to high-order mean motion resonances [11].

# 2 Aims

In the first part of the dissertation I study the libration of trojan asteroids around the point $L_4$. The number of such known asteroids in the solar system is greater than 6000. I investigate their motion using the model of the elliptic restricted three body problem (ERTBP) and the methods provided by Floquet's theorem. For stability investigation I analyse the motion of a test particle, placed in the vicinity of the point $L_4$, on the $(\mu, e)$ plane and determine the characteristic roots and exponents. From the characteristic exponents I derive the frequencies and resonances of the motion in the stable and unstable region also.

In the second part of the dissertation I investigate high-order (third and fourth order) mean motion resonances within the orbit of Jupiter, and beyond Neptune. I determine the librational regions of the resonance variable for the 8:5, 7:4, 9:5, 7:3, 5:2, 4:1 and 5:1 resonances on the semi-major axis, eccentricity plane, near the resonant semi-major axis. For the numerical investigation I use the model of the circular restricted three body problem (RTBP). The dynamical structure of the resonance is investigated using the RLI chaos detection method.

# 3 Methods

For the numerical study of the motions of trojan asteroids I applied the model of the ERTBP, and for investigating high-order resonances the model of the RTBP was used. To determine the orbits I used a fourth order Runge-Kutta scheme.

To determine the characteristic roots of the first variational equations I applied Floquet's theory. The roots and exponents were determined by using GNU Octave's built-in functions. As an alternative method to compute the frequencies I used built-in functions for fast Fourier transform in the GNU Scientific Library.

I mapped the dynamical structures of resonances with chaos indicator RLI (Relative Lyapunov Indicator, [12]).

# 4 Theses

1. I determined the characteristic roots and exponents of the first variational equations of the ERTBP numerically, on a grid covering the entire $(\mu, e)$ plane. Results are in good agreement with previous researches, determining stable and unstable regions, but are more general. [i]

2. From the imaginary parts of the exponents I determined the librational frequencies around the point $L_4$ in the stable and unstable region also. For comparison I determined the frequencies using FFT as an alternative method, the results agreed well with those obtained with Floquet's theory. I have shown, that FFT could provide the frequencies only for $\mu, e$ in the stable domain. [i]

3. Fitting the four librational frequencies in the entire stable and in one part of the unstable domain, I derived functions that provide the dependencies between the frequencies of motion around the point $L_4$ and the mass parameter and the eccentricitty. Frequencies computed using these functions are in good agreement with frequencies determined by numerical methods in the inside of the regions, however near the boundaries dividing the stable and unstable domains the agreement decays.[i]

4. I determined the resonances between the four libtrational frequencies on the $(\mu, e)$ parameter plane. I have shown that the boundaries between the stable and unstable domains correspond to 1:1 resonances of different types. These 1:1 resonances do not occur along narrow curves as suggested before, but on extended regions, that cover the entire unstable domain together. I have shown that resonances are in strong relation with the lifetime of the system. [i]

5. I investigated third and fourth order inner resonances in the Sun-Jupiter system ($\mu = 0.001$) on the semi-major axis - eccentricity plane, in the vicinity of the resonant semi-major axis. I have shown, that two librational domain appear in the case of 8:5 and 7:4 mean motion resonances, one for low and one for high eccentricities; libration of the resonance variable do not occur in the case of 13:9, 11:7 and 5:1 resonances; one extended symmetric librational domain is present on

the $(a, e)$ parameter plane for 5:2 and 4:1 resonances. I investigated the dynamical structure of resonances using the RLI chaos indicator, and showed that librational domains of the resonance variable are in good agreement with regular regions on the RLI maps. Chaotic regions appear when the orbits of the perturbing and perturbed bodies cross, and the size of chaotic region shrinks moving away form the perturbing body. [ii]

6. I investigated third and fourth order outer resonances in the Sun-Neptune system ($\mu = 0.00005$) on the semi-major axis - eccentricity plane, in the vicinity of the resonant semi-major axis. I have shown, that two librational domain appear in the case of 8:5, 7:4, 7:3, 5:2 and 5:1 mean motion resonances; one librational region corresponds to resonance 4:1 and three to resonance 9:5. The 5:2 case is very regular and symmetric. I have shown that the amplitudes of libration of the resonance variable are significantly larger than in the case of inner resonances. [ii]

# 5 Publications

## 5.1 Fundamental publications of the theses

i Rajnai R., Nagy I., Érdi B., 2014, *Frequencies and resonances around $L_4$ in the elliptic restricted three-body problem*, Monthly Notices of the Royal Astronomical Society, elfogadva, megjelenés alatt DOI: 10.1093/mnras/stu1212

ii Érdi B., Rajnai R., Sándor Zs., Forgács-Dajka E., 2012, *Stability of higher order resonances in the restricted three-body problem*, Celestial Mechanics and Dynamical Astronomy, Volume 113, pp.95-112

## 5.2 Other publications in the topic of the dissertation

- Érdi B., Forgács-Dajka E., Rajnai R., 2014, *A numerical study of capture into the 7:4 mean-motion resonance in the trans-Neptunian region*, Monthly Notices of the Royal Astronomical Society, beküldve, elbírálás alatt
- Rajnai R, Nagy I, Érdi B. *Frequncies of librational motions around the Lagrange point L4*, Publications of the Astronomy Department of the Eötvös University - PADEU, 20: p. 67. (2011)
- Rajnai R., Nagy I., Érdi B., 2010 *Features of librational motions around $L_4$*, Journal of Physics: Conference Series, Volume 218, Issue 1, id. 012018 (2010).
- Érdi B., Forgács-Dajka E., Nagy I., Rajnai R., 2009 *A parametric study of stability and resonances around $L_4$ in the elliptic restricted three-body problem*, Celestial Mechanics and Dynamical Astronomy, Volume 104, Issue 1-2, pp. 145-158

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